

sheet 1

I] Find the Laplace transform of following signals:-

a)  $f_1(t) = t e^{4t} + \sin(t-2)$

$$t \rightarrow \frac{1}{s^2} \quad \Rightarrow \quad t e^{4t} \rightarrow \frac{1}{(s-4)^2}$$

$$\sin t \rightarrow \frac{1}{s^2 + 1} \quad \Rightarrow \quad \sin(t-2) \rightarrow \frac{e^{-2s}}{s^2 + 1}$$

$$\text{Ans) } F(s) = \frac{1}{(s-4)^2} + \frac{e^{-2s}}{s^2 + 1}$$

b)  $f_2(t) = e^{-2t} \cos(t)$

$$\cos t \rightarrow \frac{s}{s^2 + 1}$$

$$e^{-2t} \cos t \rightarrow \frac{(s+2)}{(s+2)^2 + 1}$$

$$F(s) = \frac{(s+2)}{(s+2)^2 + 1}$$

$$c) f_3(t) = 3e^{-2t} - 2e^{-t} + u\left(\frac{t}{5}\right)$$

$$= \frac{3}{s+2} - \frac{2}{s+1} + \frac{1}{\underline{\underline{5s}}}$$

[2] Find the inverse Laplace transform of the following signals:-

$$a) X_1(s) = \frac{s^2 + 2}{s(s+2)(s+1)}$$

$$X_1(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1}$$

بمجرد التفكيك

$$X_1(s) = \frac{1}{s} + \frac{4}{s+2} + \frac{-3}{s+1}$$

$$x(t) = u(t) + 4e^{-2t} - 3e^{-t}$$

$$b) X_2(s) = \frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$$\frac{1}{s} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$$A=1$$

نقوم بتوصيد المعادلات

$$\frac{1}{s(s^2+1)} = \frac{As^2 + A + Bs^2 + Cs}{s(s^2+1)}$$

$$As^2 + A + Bs^2 + Cs = 1$$

$$s^2 + Bs^2 + Cs = 0$$

$$C \cdot s^2 \Rightarrow 1 + B = 0 \Rightarrow B = -1$$

$$C \cdot s \rightarrow C = 0$$

$$X_2(s) = \frac{1}{s} - \frac{s}{s^2+1}$$

$$x(t) = u(t) - \cos(t)$$

$$c) X_3(s) = \frac{1}{s^2+4s-5}$$

نحسب جذور المعادلة  $\Leftarrow$

$$X_3(s) = \frac{1}{(s-1)(s+5)}$$

$$X_3(s) = \frac{A}{s-1} + \frac{B}{s+5} = \frac{1/6}{s-1} + \frac{-1/6}{s+5}$$

$$X_3(t) = \frac{1}{6} e^t - \frac{1}{6} e^{-5t}$$



3 Determine the initial and final values of the signals that have the following Laplace transforms:-

$$a) X_1(s) = \frac{3s^2 + 1}{4s^3 + 3s^2 + 2s}$$

$$X(0) = \lim_{s \rightarrow \infty} \frac{3s^2 + 1}{4s^3 + 3s^2 + 2s}$$

بالقسمة على أكبر أس في المقام

$$X(0) = \lim_{s \rightarrow \infty} \frac{3 \frac{s^2}{s^3} + \frac{1}{s^3}}{4 + 3 \frac{s^2}{s^3} + 2 \frac{s}{s^3}} = \boxed{0}$$

$$X(\infty) = \lim_{s \rightarrow 0} \frac{s(3s^2 + 1)}{4s^3 + 3s^2 + 2s}$$

$$= \lim_{s \rightarrow 0} \frac{3s^2 + 1}{4s^2 + 3s + 2} = \boxed{\frac{1}{2}}$$

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$$b) X_2(s) = \frac{3s^2 + 1}{4s^3 + 3s^2 + 2s + 1}$$

$$X(0) = \lim_{s \rightarrow \infty} \frac{3s^2 + 1}{4s^3 + 3s^2 + 2s + 1}$$

$$= \lim_{s \rightarrow \infty} \frac{3 \frac{1}{s} + \frac{1}{s^3}}{4 + 3 \frac{1}{s} + 2 \frac{1}{s^2} + \frac{1}{s^3}} = 0$$

$$X(\infty) = \lim_{s \rightarrow 0} \frac{s(3s^2 + 1)}{4s^3 + 3s^2 + 2s + 1}$$

4 Find the impulse response of the following zero initial conditions differential equations:-

$$a) \ddot{y}(t) + 6 \dot{y}(t) + 8 y(t) = 3 \dot{u}(t) + 2u(t)$$

$$\boxed{\text{sol}} \quad u(t) = \delta(t)$$

→ Laplace transform ~~and~~ assume zero initial value.

$$s^2 Y(s) + 6s Y(s) + 8 Y(s) = 3s U(s) + 2U(s)$$

$$(s^2 + 6s + 8) Y(s) = U(s) (3s + 2)$$

$$\frac{Y(s)}{U(s)} = \frac{3s + 2}{s^2 + 6s + 8} \rightarrow \text{transfer function.}$$

$$U(s) = 1$$

$$Y(s) = \frac{3s + 2}{s^2 + 6s + 8} = \frac{A}{s + 4} + \frac{B}{s + 2}$$

$$Y(s) = \frac{5}{s + 4} - \frac{2}{s + 2}$$

$$\therefore y(t) = 5e^{-4t} - 2e^{-2t}$$



$$b) \ddot{y}(t) + 3\dot{y}(t) + 2y(t) = \ddot{u}(t) + 2\dot{u}(t) + 2u(t)$$

sol

→ Laplace transform & zero initial values

$$s^2 Y(s) + 3s Y(s) + 2 Y(s) = s^2 U(s) + 2s U(s) + 2 U(s)$$

$$(s^2 + 3s + 2) Y(s) = (s^2 + 2s + 2) U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 2s + 2}{s^2 + 3s + 2} \Rightarrow U(s) = 1$$

$$Y(s) = \frac{s^2 + 2s + 2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$Y(s) = \frac{1}{s+1} - \frac{2}{s+2}$$

$$\frac{4-4+2}{-1}$$

$$y(t) = e^{-t} - 2e^{-2t}$$

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